DIFFERENTIAL SUBGRID MODELING OF TURBULENCE

IN THE CALCULATION OF JET FLOWS

M. A. Gorokhovskii and A. A. Golovko UDC 535.517.4

The approach of Yu. L. Klimontovich to supergrid description of turbulent motions is developed. An illustration of its effectiveness is presented.

1. The second moments of pulsational quantities, obtained by time-averaging convective terms in the starting transport equations, are as a rule closed with the help of gradient relations, in which the coefficients of turbulent diffusion of momentum and heat are written in accordance with the well-known semiempirical models $k—L—\langle T'^2\rangle$, $k—\varepsilon—\langle T'^2\rangle$ [1]:

$$\nu_\text{т} = C'_\mu k^{1/2}L, \quad \nu_\text{т} = C_\mu \frac{k^2}{\varepsilon}, \quad a_\text{т} = \frac{\nu_\text{т}}{\text{Pr}_\text{т}}.$$

It is obvious that here the question of the spatial scales of fluctuations of the random quantities is ignored. In addition, it is not clear how on the basis of this approach the turbulence is modelled, if the mathematical description of the flow is given in Lagrangian variables, as, for example, in the ALE method [2]. There is also a different method — effective regularization of transport equations in the presence of gas-thermodynamic fluctuations [3-5]. This method consists of the following. Langevin random sources, whose existence is postulated, are introduced into the equations. The second moments of the Langevin forces are δ-correlated in both time and space, and the intensity in the relations for the moments of these forces is expressed in terms of one-time correlation functions of the fluctuations of the velocity and temperature. Next, replacing the δ function by a Gaussian distribution and carrying out some calculations, the following expressions were obtained in [4] for the "diffusion" coefficients:

$$\nu_\text{eff} = \nu \left[1 + \frac{l_\text{ph}^2 (u_{i,j})^2}{\langle (\delta u'_{V_\text{ph}})^2 \rangle} \right], \tag{1}$$

$$a_\text{eff} = a \left[1 + \frac{l_\text{ph}^2 (\text{grad} \langle T \rangle)^2}{\langle (\delta T'_{V_\text{ph}})^2 \rangle} \right], \tag{2}$$

where $\ell_\text{ph} (\ell_\text{ph} \ll L)$ is a physically small parameter. Within the volume $V_\text{ph} (V_\text{ph} \sim \ell^3_\text{ph})$ small details of the motion, which cannot be observed, are excluded; $u_{i,j}$ is the strain-rate tensor; $\delta u'_{V_\text{ph}}$, $\delta T'_{V_\text{ph}}$ are the deviations of the velocity and temperature averaged over the volume V_ph from the average phase values.

2. In the relation (1) the variance $0.5 \langle(\delta u'_{V_\text{ph}})^2\rangle$ is the kinetic energy of the subgrid turbulence k, while the expression $\nu(u_{i,j})^2$ is identical to the expression for the rate of viscous dissipation ε. The variance $\langle (\delta T'_{V_\text{ph}})^2 \rangle$ in (2) can be represented as the square of the subgrid pulsations of the temperature $\langle T'^2\rangle$. In so doing it is assumed that k, ε, and $\langle T'^2\rangle$ obey the well-known differential transport equations of the $k — \varepsilon — \langle T'^2 \rangle$ model with the use of the standard set of constants [3, 6-8].

The size of the control cell of the finite-difference grid is chosen as the small scale ℓ_ph: $\Delta \sim \ell_\text{ph} (\Delta \ll L)$. In this case the relations (1) and (2) assume the form

$$\nu_\text{eff} = \nu + \Delta^2 \frac{\varepsilon}{2k},$$

S. M. Kirov Kazakh State University, Alma-Ata. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 57, No. 1, pp. 41-44, July, 1989. Original article submitted February 18, 1988.

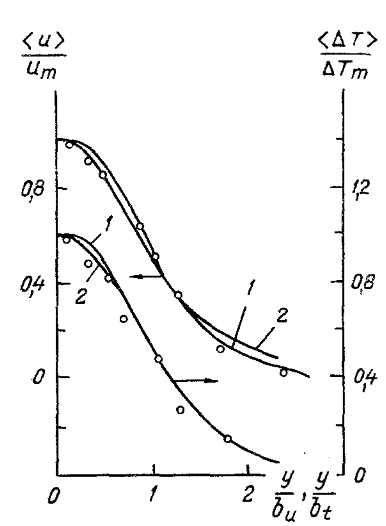

Fig. 1

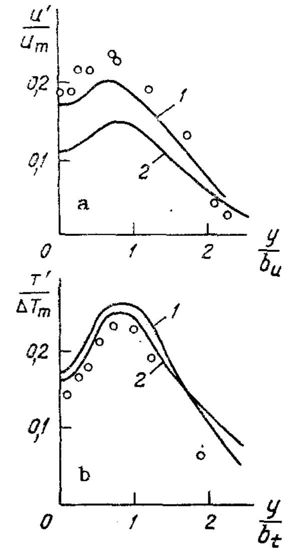

Fig. 2

Fig. 1. Profiles of the average velocity and the average temperature differential: 1) k-ε-$\langle T'^2\rangle$; 2) k-ε-$\langle T'^2\rangle$-Δ; the dots are the experimental values of [9].

Fig. 2. The distribution of velocity and temperature pulsations. The notation is the same as in Fig. 1.

$$a_{\text{eff}} = a\left[1 + \frac{\Delta^2(\text{grad}\,\langle T\rangle)^2}{\langle T'^2\rangle}\right],$$

and there is no need to introduce the turbulent Prandtl number.

We note that on the basis of this approach it is possible to construct a model of turbulence of the type k—Δ—$\langle T'^2\rangle$ containing two differential transport equations: for the kinetic turbulent energy and for pulsations of the temperature with effective diffusion coefficients taken directly in the form (1) and (2). The dissipative terms in the equations for k and $\langle T'^2\rangle$ can be obtained from the assumption that the Kolmogorov microscale characterizing small vortices is "artificially" expanded up to the dimensions of the control cell of the finite-difference grid: ν_T^3/Δ^4 and a_T^3/Δ^4.

3. The relations presented were employed to solve the problem of the propagation of a flat, submerged, turbulent jet of air. The published [9] experimental data were compared with the data computed using the standard $k-L-\langle T'^2\rangle$ and $k-\varepsilon-\langle T'^2\rangle$ models and using the $k-\Delta-\langle T'^2\rangle$, $k-\varepsilon-\langle T'^2\rangle-\Delta$ models. The comparisons of the experimental distributions of pulsations with calculations according to the $k-L-\langle T'^2\rangle$ and the "supergrid" $k-\Delta-\langle T'^2\rangle$ models of turbulence using the standard constants [6] turned out to be unsatisfactory, and for this reason the results of calculations based on the $k-\varepsilon-\langle T'^2\rangle$ model and the "supergrid" $k-\varepsilon-\langle T'^2\rangle-\Delta$ model modified according to Yu. L. Klimontovich are demonstrated below.

As one can see from Fig. 1, both models predict the experimental data with a high degree of accuracy.

The results of comparison of the dimensionless pulsations of the velocities in the transverse section of the jet are shown in Fig. 2a. The level of pulsations computed according to the standard $k-\varepsilon-\langle T'^2\rangle$ model with the help of the relation $(\langle u'^2\rangle)^{1/2}\sim\left(\frac{2}{3}k\right)^{1/2}$ agrees well with experiment, while the values of the pulsations of the velocity, calculated in the same manner using the $k-\varepsilon-\langle T'^2\rangle-\Delta$ model, are somewhat lower than the experimental values.

At the same time both models permit obtaining good agreement with the experimental distributions of the temperature pulsations (Fig. 2b).

4. In this work the turbulence is modeled with the help of "dissipative" methods developed in [4, 5] for describing thermodynamically irreversible processes. We shall give some arguments confirming that they are analogous to the methods of subgrid modeling of turbulence [10-12]. In the inertial interval the gradient of the velocity for vortices of size λ is of the order of $\varepsilon^{1/3}\lambda^{-2/3}$, where ε is the dissipation of energy per unit time and unit mass. The velocity gradient increases as λ decreases and becomes so large that molecular viscosity cannot be neglected. This occurs when $\lambda \sim \eta$, where $\eta = (\nu^3/\varepsilon)^{1/4}$ is the Kolmogorov microscale. The velocity gradient $\partial u_i/\partial x_j$ in the smallest vortices should have a magnitude that is required for viscous dissipation, i.e., $2\nu u_{i,j}u_{i,j} = \nu^3/\eta^4$. Further, if it is assumed that the minimum scale resolvable for vortices, within which pulsations are neglected, is the size of the cell of the finite-difference grid, then the well-known simple subgrid model of Deardorff is obtained: $\nu_T \sim \Delta^2(2u_{i,j}u_{i,j})^{1/2}$, where the "dissipative" scale is artificially expanded up to the sizes of the grid cell [12]. A similar approach is studied in this work. The effect of the δ-correlated Langevin sources is "smeared," over some physical scale, equal to the size of the control cell of the grid. In this case we can also say that the dissipating turbulent vortices are artificially expanded to the minimum resolvable spatial scale. Here the well-known expression $\nu_T = C_\mu k^2/\varepsilon$ is not employed, but rather the "subgrid" formula $\nu_T = \Delta^2\varepsilon/2k$, which does not contain C_μ, is employed. A similar approach was specially tested in calculations of jets in order to demonstrate its effectiveness together with the effectiveness of the standard $K-\varepsilon-\langle T'^2\rangle$ model well-known for parabolic flows. The possibilities of such "supergrid, irrational" description of turbulent flows will be demonstrated by further investigations, associated with the search for an optimal collection of model constants, the absence of which could be a justification for the discrepancy between the computed and experimental data presented in Fig. 2a.

NOTATION

u, velocity along the flow; T, temperature; $\Delta T = T - T_\infty$, temperature differential; u' and T', velocity and temperature pulsations; ν and a, coefficients of kinematic viscosity and thermal diffusivity; Pr, Prandtl number; k, kinetic energy of turbulence; ε, rate of dissipation of k; y, transverse coordinate; L, geometric dimensions of the flow; and b, half-width. The indices ∞ and m indicate that the value of the quantity in the exterior flow or on the axial line is to be taken; the indices u, t and T indicate that the quantity refers to the velocity, the temperature, or the turbulent characteristic, respectively.

LITERATURE CITED

1. B. E. Launder and D. B. Spalding, ZAMM, 56, 219-221 (1976).
2. S. Hert, Numerical Methods in Fluid Mechanics [Russian translation], Moscow (1973), pp. 156-164.
3. L. D. Landau and E. M. Lifshits, "On hydrodynamic fluctuations," in: Collected Works of L. D. Landau [in Russian], Vol. 2, Moscow (1969), pp. 376-378.
4. Yu. L. Klimontovich, Statisticl Physics [in Russian], Moscow (1982).
5. I. Prigozhine, From Being to Becoming [Russian translation], Moscow (1985).
6. V. Kal'man (ed.), Computational Methods for Turbulent Flows [in Russian], Moscow (1984), pp. 227-268.
7. D. B. Spalding, Chem. Eng. Sci., 26, 95-107 (1971).
8. V. I. Golovichev, M. A. Gorokhovskii, and V. P. Kashkarov, Inzh.-Fiz. Zh., 44, No. 6, 933-942 (1983).
9. B. F. Ramaprian and M. S. Chandrasekara, Theoretical Foundations of Engineering Calculations [in Russian], Vol. 102, Moscow (1985), No. 2, pp. 177-185.
10. J. W. Deardorff, J. Fluid Mech., 41, 453-467 (1970).
11. J. W. Deardorff, J. Comput. Phys., 7, No. 1, 120-133 (1971).
12. J. D. Ramshaw, Informal Report, Los Alamos Scientific Laboratory Report LA-7955-MS (1979).